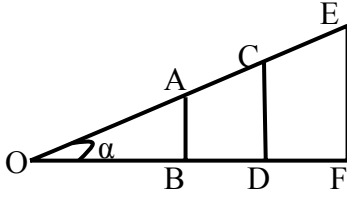


Temel Trigonometrik bağıntılar



Bir dik üçgende aşağıdaki bağıntıların varlığı tarih öncesinden biliniyordu.

$$\frac{AB}{OB} = \frac{CD}{OD} = \frac{EF}{OF}, \quad \frac{AB}{OA} = \frac{CD}{OC} = \frac{EF}{OE}, \quad \frac{OB}{OA} = \frac{OD}{OC} = \frac{OF}{OE}$$

Temel trigonometrik bağıntılar aşağıdaki şekildedir.

$$\sin \alpha = \frac{AB}{OA} = \frac{CD}{OC} = \frac{EF}{OE},$$

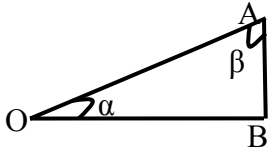
$$\cos \alpha = \frac{OB}{OA} = \frac{OD}{OC} = \frac{OF}{OE},$$

$$\tan \alpha = \frac{AB}{OB} = \frac{CD}{OD} = \frac{EF}{OF}$$

Pratikte az kullanılan $\cot \alpha$, $\sec \alpha$, $\csc \alpha$ tanımları da aşağıdaki gibidir.

$$\cot \alpha = \frac{1}{\tan \alpha} = \frac{OB}{AB}, \quad \csc \alpha = \frac{1}{\sin \alpha} = \frac{OA}{AB}, \quad \sec \alpha = \frac{OA}{OB},$$

Ayrıca



$$\cos \beta = \frac{AB}{OA}, \quad \sin \beta = \frac{OB}{OA}, \quad \cot \beta = \frac{AB}{OB}$$

. $\alpha + \beta = 90^\circ$ olduğundan.

$$\sin(x) = \cos(90-x), \quad \cos(x) = \sin(90-x), \quad \tan(x) = \cot(90-x), \quad \cot(x) = \tan(90-x)$$

olduğu açıktır.

Bazı kritik açılar,

$\alpha=0$ olursa $AB=0$, ve $OA=OB$ olur.



$$\alpha=0, \quad AB=0, \quad OB=OA$$

$$\sin 0 = \frac{AB}{OA} = \frac{0}{OA} = 0$$

$$\cos 0 = \frac{OB}{OA} = \frac{OA}{OA} = 1,$$

$$\tan \alpha = \frac{AB}{OB} = \frac{0}{OB} = 0$$

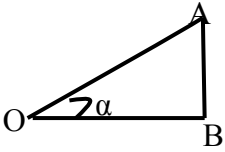
$\alpha=90^0$ olursa $OB=0$, ve $OA=AB$ olur.



$$\sin 90^0 = \frac{AB}{OA} = \frac{OA}{OA} = 1 \quad \cos 90^0 = \frac{OB}{OA} = \frac{0}{OA} = 0,$$

$$\tan 90^0 = \frac{AB}{0} = \infty$$

$\alpha=45^0$ olursa $OB=AB$, $OA=\sqrt{OB^2 + AB^2} = \sqrt{AB^2 + AB^2} = \sqrt{2} AB$



$$\sin 45^0 = \frac{AB}{OA} = \frac{AB}{\sqrt{2}AB} = \frac{1}{\sqrt{2}} = 0.707$$

$$\cos 45^0 = \frac{OB}{OA} = \frac{AB}{\sqrt{2}AB} = \frac{1}{\sqrt{2}} = 0.707$$

$$\tan 45^0 = \frac{AB}{OB} = \frac{AB}{AB} = 1$$

Geçiş Bağlılıları

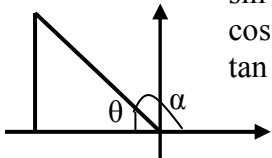
Sinus ve cosinus tanımlarını kullanarak

$$\frac{\sin \alpha}{\cos \alpha} = \frac{\frac{AB}{OA}}{\frac{OB}{OA}} = \frac{AB}{OB} = \tan \alpha \quad \text{bağlılısını elde ederiz.}$$

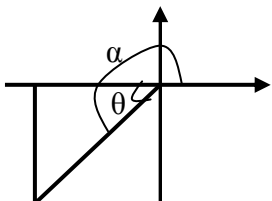
Pisagor bağılılısını kullanarak:

$$\sin^2 \alpha + \cos^2 \alpha = \left(\frac{AB}{OA}\right)^2 + \left(\frac{OB}{OA}\right)^2 = \frac{(AB)^2 + (OB)^2}{(OA)^2} = \frac{(OA)^2}{(OA)^2} = 1$$

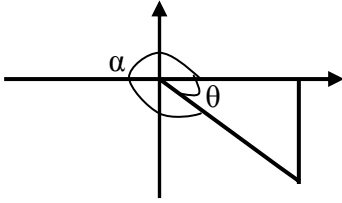
90^0 den büyük açıların sinüs, cosinus, tanjantları:



$$\begin{aligned} \sin \alpha &= \sin \theta = \sin(180^0 - \alpha) \\ \cos \alpha &= -\cos \theta = -\cos(180^0 - \alpha) \\ \tan \alpha &= -\tan \theta = -\tan(180^0 - \alpha) \end{aligned}$$



$$\begin{aligned} \sin \alpha &= -\sin \theta = -\sin(\alpha - 180^0) \\ \cos \alpha &= -\cos \theta = -\cos(\alpha - 180^0) \\ \tan \alpha &= -\tan \theta = \tan(\alpha - 180^0) \end{aligned}$$



$$\begin{aligned}\sin \alpha &= -\sin \theta = -\sin(360 - \alpha) \\ \cos \alpha &= \cos \theta = \cos(360 - \alpha) \\ \tan \alpha &= -\tan \theta = -\tan(360 - \alpha)\end{aligned}$$

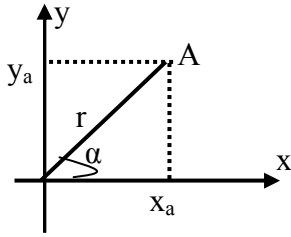
Pratik yol

a) α Açısını x-y koordinat sisteminde çiz.

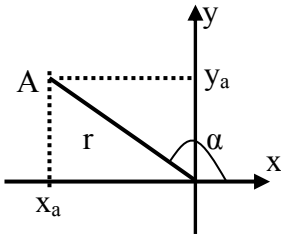
b) $\sin \alpha = x_a/r$, $\cos \alpha = y_a/r$, $\tan \alpha = x_a/y_a$

$$r = \sqrt{x_a^2 + y_a^2}$$

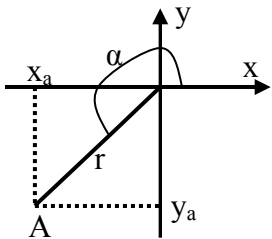
burada x_a ve y_a işaretleri ile dikkate alınmalıdır.



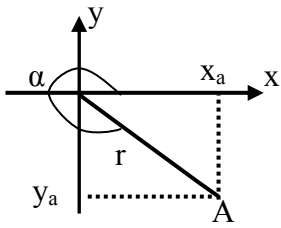
$$\begin{aligned}x_a > 0, \quad y_a > 0 \\ \sin \alpha &= \frac{x_a}{r} > 0, & \cos \alpha &= \frac{y_a}{r} > 0, \\ \tan \alpha &= \frac{x_a}{y_a} > 0,\end{aligned}$$



$$\begin{aligned}x_a < 0, \quad y_a > 0 \\ \sin \alpha &= \frac{x_a}{r} < 0, & \cos \alpha &= \frac{y_a}{r} > 0, \\ \tan \alpha &= \frac{x_a}{y_a} < 0,\end{aligned}$$



$$\begin{aligned}x_a < 0, \quad y_a < 0 \\ \sin \alpha &= \frac{x_a}{r} < 0, & \cos \alpha &= \frac{y_a}{r} < 0, \\ \tan \alpha &= \frac{x_a}{y_a} > 0,\end{aligned}$$



$$x_a < 0, \quad y_a > 0$$

$$\sin \alpha = \frac{y_a}{r} > 0,$$

$$\cos \alpha = \frac{x_a}{r} < 0,$$

$$\tan \alpha = \frac{y_a}{x_a} < 0,$$

----- $90^\circ < \alpha < 180^\circ$ -----

$$\sin(100^\circ) = \sin(180^\circ - 100^\circ) = \sin(80^\circ) = 0.9848$$

$$\cos(100^\circ) = -\cos(180^\circ - 100^\circ) = -\cos(80^\circ) = -0.1736$$

$$\tan(100^\circ) = -\tan(180^\circ - 100^\circ) = -\tan(80^\circ) = -5.6713$$

----- $180^\circ < \alpha < 270^\circ$ -----

$$\sin(200^\circ) = -\sin(200^\circ - 180^\circ) = -\sin(20^\circ) = -0.3420$$

$$\cos(200^\circ) = -\cos(200^\circ - 180^\circ) = -\cos(20^\circ) = -0.9397$$

$$\tan(200^\circ) = \tan(200^\circ - 180^\circ) = \tan(20^\circ) = 0.364$$

----- $270^\circ < \alpha < 360^\circ$ -----

$$\sin(300^\circ) = -\sin(360^\circ - 300^\circ) = -\sin(60^\circ) = -0.8660$$

$$\cos(300^\circ) = \cos(360^\circ - 300^\circ) = \cos(60^\circ) = 0.5$$

$$\tan(300^\circ) = -\tan(360^\circ - 300^\circ) = -\tan(60^\circ) = -1.732$$

	0°		90°		180°		270°		$360^\circ = 0^\circ$
sin	0	+	1	+	0	-	-1	-	0
cos	1	+	0	-	-1	-	0	+	1
tan	0	+	∞	-	0	+	$-\infty$	-	0

