

Formullerin türetilmesi ile ilgili aciklamalar ek sayfalardaki esitlik numaralarinda verilmiştir.

$\sin^2x + \cos^2x = 1$		$\cos q = 2\cos^2 \frac{q}{2} - 1$	(21)
$\cos(x+y) = \cos x \cos y - \sin x \sin y$	(1)	$\cos q = 1 - 2\sin^2 \frac{q}{2}$	(22)
$\sin(x+y) = \sin x \cos y + \sin y \cos x$	(2)	$\sin q = 2 \sin \frac{q}{2} \cos \frac{q}{2}$	(23)
$\sin(-x) = -\sin x$	(3)		
$\cos(-x) = \cos(x)$	(4)	$\sin^3 q = -\frac{1}{4} \sin(3q) + \frac{3}{4} \sin q$	(34)
$\tan(-x) = -\tan(x)$	(4.a)	$\sin(3q) = -4\sin^3 q + 3\sin q$	(35)
$\cos(x-y) = \cos x \cos y + \sin x \sin y$	(5)	$\cos^3 q = \frac{1}{4} \cos 3q + \frac{3}{4} \cos q$	(36)
$\sin(x-y) = \sin x \cos y - \sin y \cos x$	(6)	$\cos 3q = 4 \cos^3 q - 3 \cos q$	(37)
$\cos x \cos y = \frac{1}{2} [\cos(x+y) + \cos(x-y)]$	(7)	$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$	(41)
$\sin x \sin y = \frac{1}{2} [\cos(x-y) - \cos(x+y)]$	(8)	$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$	(42)
$\sin x \cos y = \frac{1}{2} [\sin(x+y) + \sin(x-y)]$	(9)	$\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$	(43)
$\cos(x+90) = -\sin x$	(10)	$\tan(q) = \frac{2 \tan \frac{q}{2}}{1 - \tan^2 \frac{q}{2}}$	(44)
$\cos(x-90) = \sin x$	(11)		
$\sin(x+90) = \cos x$	(12)	$\tan x = \frac{\sqrt{1 - \cos^2 x}}{\cos x}$	(51)
$\sin(x-90) = -\cos x$	(13)	$\tan x = \frac{\sin x}{\sqrt{1 - \sin^2 x}}$	
$\cos(90-q) = \sin q$	(14)		
$\sin(90-q) = \cos q$	(15)	$\sin x = \frac{\tan x}{\sqrt{1 + \tan^2 x}}$	(52)
$\cos(2x) = 2\cos^2 x - 1$	(16)	$\cos x = \frac{1}{\sqrt{1 + \tan^2 x}}$	(53)
$\cos^2 x = \frac{1}{2} [1 + \cos(2x)]$	(17)		
$\cos(2x) = 1 - 2\sin^2 x$	(18)	$\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$	(55)
$\sin^2 x = \frac{1}{2} [1 - \cos(2x)]$	(19)	$\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$	(57)
$\sin 2x = 2 \sin x \cos x$	(20)		

Formullerin Turetilmesi

$$\sin^2 x + \cos^2 x = 1$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y \quad (1)$$

$$\sin(x+y) = \sin x \cos y + \sin y \cos x \quad (2)$$

$$\sin(-x) = -\sin x \quad (3)$$

$$\cos(-x) = \cos(x) \quad (4)$$

$$\tan(-x) = -\tan(x) \quad (4.a)$$

x	0	30	45	60	90
sinx	0	0.5	0.707	0.8	1
cosx	1	0.8	0.707	0.5	0

Diger formuller yukaridaki formullerde turetilerek cikarilabilir

(1) de y yerine -y yaz ve 3,4 bagintilarini kullan

$$\cos(x-y) = \cos x \cos(-y) - \sin x \sin(-y)$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y \quad (5)$$

(2) de y yerine -y yaz ve 3,4 bagintilarini kullan

$$\sin(x-y) = \sin x \cos(-y) - \sin x \cos(-y)$$

$$\sin(x-y) = \sin x \cos y - \sin y \cos x \quad (6)$$

(1) ve (5) i topla

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

+

$$\cos(x+y) + \cos(x-y) = 2 \cos x \cos y$$

$$\cos x \cos y = \frac{1}{2} [\cos(x+y) + \cos(x-y)] \quad (7)$$

(5) den (1) i cikar

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

-

$$\cos(x-y) - \cos(x+y) = 2 \sin x \sin y$$

$$\sin x \sin y = \frac{1}{2} [\cos(x-y) - \cos(x+y)] \quad (8)$$

(2) ve (6) yi topla

$$\sin(x+y) = \sin x \cos y + \sin y \cos x \quad (2)$$

$$\sin(x-y) = \sin x \cos y - \sin y \cos x \quad (6)$$

+

$$\sin(x+y) + \sin(x-y) = 2 \sin x \cos y$$

$$\sin x \cos y = \frac{1}{2} [\sin(x+y) + \sin(x-y)] \quad (9)$$

(1) de x=90 koy

$$\cos(x+90) = \cos x \cos 90 - \sin x \sin 90$$

$$\cos(x+90) = -\sin x \quad (10)$$

(1) de $x=-90$ koy

$$\cos(x-90)=\cos x \cos(-90) - \sin x \sin(-90)$$

$$\cos(x-90)=\sin x \quad (11)$$

(2) de $x=90$ koy

$$\sin(x+90)=\sin x \cos 90 + \sin 90 \cos x$$

$$\sin(x+90)=\cos x \quad (12)$$

(2) de $x=-90$ koy

$$\sin(x-90)=\sin x \cos(-90) + \sin(-90) \cos x$$

$$\sin(x-90)=-\cos x \quad (13)$$

(1) de $x=90, y=-q$ koy

$$\cos(90-q)=\cos 90 \cos(-q) - \sin 90 \sin(-q)$$

$$\cos(90-q)=\sin q \quad (14)$$

(2) de $x=90, y=-q$ koy

$$\sin(90-q)=\sin 90 \cos(-q) + \cos 90 \sin(-q)$$

$$\sin(90-q)=\cos q \quad (15)$$

(1) de $y=x$ koy

$$\cos(x+x)=\cos x \cos x - \sin x \sin x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\sin^2 x = 1 - \cos^2 x \text{ koy}$$

$$\cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

$$\cos(2x) = 2\cos^2 x - 1 \quad (16)$$

veya

$$\cos^2 x = \frac{1}{2} [1 + \cos(2x)] \quad (17)$$

(1) de $y=x$ koy

$$\cos(x+x)=\cos x \cos x - \sin x \sin x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos^2 x = 1 - \sin^2 x \text{ koy}$$

$$\cos^2 x - \sin^2 x = 1 - \sin^2 x - \sin^2 x = 1 - 2\sin^2 x$$

$$\cos(2x) = 1 - 2\sin^2 x \quad (18)$$

veya

$$\sin^2 x = \frac{1}{2} [1 - \cos(2x)] \quad (19)$$

(2) de $y=x$ koy

$$\sin(x+x)=\sin x \cos x + \sin x \cos x = 2 \sin x \cos x$$

$$\sin 2x = 2 \sin x \cos x \quad (20)$$

(16) ve (17) de da $q=2x$ koy

$$\cos q = 2\cos^2 \frac{q}{2} - 1 \quad (21)$$

$$\cos q = 1 - 2\sin^2 \frac{q}{2} \quad (22)$$

(20) de $q=2x$ koy

$$\sin q = 2 \sin \frac{q}{2} \cos \frac{q}{2} \quad (23)$$

	90	180	270	-90	-180
sinx	1	0	-1	-1	0
cosx	0	-1	0	0	-1

$$\sin^3 q = \sin q \sin^2 q$$

(19) bagintisinde verilen $\sin^2 q$ yerine koy

$$\sin^3 q = \sin q \frac{1}{2} [1 - \cos(2q)] = \frac{1}{2} [\sin q - \sin q \cos(2q)]$$

(9) bagintisinde $x=q, y=2q$ koy

$$\sin q \cos(2q) = \frac{1}{2} [\sin(q+2q) + \sin(q-2q)]$$

$$= \frac{1}{2} [\sin(3q) - \sin(q)]$$

$$\sin^3 q = \frac{1}{2} [\sin q - \frac{1}{2} [\sin(3q) - \sin q]]$$

$$= \frac{1}{2} \sin q - \frac{1}{4} \sin(3q) + \frac{1}{4} \sin q$$

$$\sin^3 q = -\frac{1}{4} \sin(3q) + \frac{3}{4} \sin q \quad (34)$$

veya

$$\sin(3q) = -4\sin^3 q + 3\sin q \quad (35)$$

$$\cos^3 q = \cos q \cos^2 q$$

(17) bagintisinde verilen $\cos^2 q$ yerine degerini koy

$$\cos^3 q = \cos q \frac{1}{2} [1 + \cos(2q)] = \frac{1}{2} [\cos q + \cos q \cos(2q)]$$

(7) bagintisinde $x=q, y=2q$ koy

$$\cos q \cos 2q = \frac{1}{2} [\cos(q+2q) + \cos(q-2q)]$$

$$= \frac{1}{2} [\cos(3q) + \cos q]$$

$$\cos^3 q = \frac{1}{2} [\cos q + \frac{1}{2} [\cos(3q) + \cos q]]$$

$$= \frac{1}{4} \cos 3q + \frac{3}{4} \cos q$$

$$\cos^3 q = \frac{1}{4} \cos 3q + \frac{3}{4} \cos q \quad (36)$$

$$\cos 3q = 4 \cos^3 q - 3 \cos q \quad (37)$$

$$\tan(x+y) = \frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin x \cos y + \sin y \cos x}{\cos x \cos y - \sin x \sin y}$$

payi ve paydayi $\cos x \cos y$ ifadesine bolelim

$$= \frac{\frac{\sin x \cos y + \sin y \cos x}{\cos x \cos y}}{\frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y}} = \frac{\frac{\sin x \cos y}{\cos x \cos y} + \frac{\sin y \cos x}{\cos x \cos y}}{\frac{\cos x \cos y}{\cos x \cos y} - \frac{\sin x \sin y}{\cos x \cos y}}$$

$$\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y} = \frac{\tan x + \tan y}{1 - \frac{\sin x \sin y}{\cos x \cos y}} = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \quad (41)$$

$\tan(-x) = -\tan(x)$ oldugu dikkate alinirsa

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} \quad (42)$$

(41) de $y=x$ koy

$$\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x} \quad (43)$$

(43) de $q=2x$ koy

$$\tan(q) = \frac{2 \tan \frac{q}{2}}{1 - \tan^2 \frac{q}{2}} \quad (44)$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\sin x}{\sqrt{1 - \sin^2 x}} = \frac{\sqrt{1 - \cos^2 x}}{\cos x} \quad (51)$$

$$\tan x = \frac{\sin x}{\sqrt{1 - \sin^2 x}} \Rightarrow \tan^2 x = \frac{\sin^2 x}{1 - \sin^2 x}$$

$$\tan^2 x (1 - \sin^2 x) = \sin^2 x$$

$$\tan^2 x - \tan^2 x \sin^2 x = \sin^2 x$$

$$\tan^2 x - \tan^2 x \sin^2 x - \sin^2 x = 0$$

$$- \tan^2 x \sin^2 x - \sin^2 x = - \tan^2 x$$

$$- \sin^2 x (\tan^2 x + 1) = - \tan^2 x$$

$$\sin^2 x = \frac{\tan^2 x}{1 + \tan^2 x} \Rightarrow \sin x = \frac{\tan x}{\sqrt{1 + \tan^2 x}} \quad (52)$$

$$\tan x = \frac{\sqrt{1 - \cos^2 x}}{\cos x} \Rightarrow \tan^2 x = \frac{1 - \cos^2 x}{\cos^2 x}$$

$$\tan^2 x \cos^2 x = 1 - \cos^2 x$$

$$\cos^2 x (1 + \tan^2 x) = 1$$

$$\cos^2 x = \frac{1}{1 + \tan^2 x} \Rightarrow \cos x = \frac{1}{\sqrt{1 + \tan^2 x}} \quad (53)$$

(23) den

$$\sin q = 2 \sin \frac{q}{2} \cos \frac{q}{2}$$

(52)ve (53) den

$$\sin \frac{x}{2} = \frac{\tan \frac{x}{2}}{\sqrt{1 + \tan^2 \frac{x}{2}}} \quad \cos \frac{x}{2} = \frac{1}{\sqrt{1 + \tan^2 \frac{x}{2}}}$$

Ucu birlestirilirse

$$\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2} = 2 \frac{\tan \frac{x}{2}}{\sqrt{1 + \tan^2 \frac{x}{2}}} \frac{1}{\sqrt{1 + \tan^2 \frac{x}{2}}} \quad \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \quad (54)$$

(21) ve (53) den

$$\cos x = 2 \cos^2 \frac{x}{2} - 1 = 2 \left(\frac{1}{\sqrt{1 + \tan^2 \frac{x}{2}}} \right)^2 - 1$$

$$\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \quad (55)$$

(54) ve (55) icin baska bir yol.

(53) ve (44) birlestirilirse

$$\cos x = \frac{1}{\sqrt{1 + \tan^2 x}} = \frac{1}{\sqrt{1 + \left(\frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}} \right)^2}}$$

basitlestirmek icin $q = \tan(x/2)$ tanimi yapalim.

$$= \frac{1}{\sqrt{1 + \left(\frac{2q}{1 - q^2} \right)^2}} = \frac{1}{\sqrt{1 + \frac{4q^2}{1 - 2q^2 + q^4}}} = \frac{1}{\sqrt{\frac{1 - 2q^2 + q^4 + 4q^2}{1 - 2q^2 + q^4}}}$$

$$= \frac{1}{\sqrt{\frac{1 + 2q^2 + q^4}{1 - 2q^2 + q^4}}} = \frac{1}{\sqrt{\frac{(1 + q^2)^2}{(1 - q^2)^2}}} = \frac{1 - q^2}{1 + q^2} = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \quad (56)$$

(52) ve (44) birlestirilirse

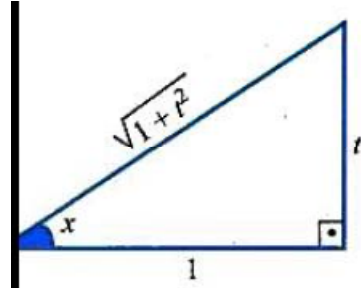
$$\sin x = \frac{\tan x}{\sqrt{1 + \tan^2 x}} = \frac{\frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}}}{\sqrt{1 + \left(\frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}} \right)^2}}$$

$q = \tan(x/2)$ tanimi yapalım.

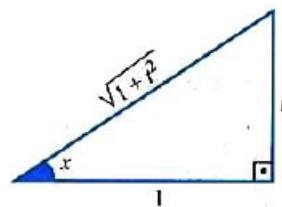
$$\begin{aligned} &= \frac{\frac{2q}{1-q^2}}{\sqrt{1 + \left(\frac{2q}{1-q^2} \right)^2}} = \frac{\frac{2q}{1-q^2}}{\sqrt{1 + \frac{4q^2}{1-2q^2+q^4}}} = \frac{\frac{2q}{1-q^2}}{\sqrt{\frac{1-2q^2+q^4}{1-2q^2+q^4} + \frac{4q^2}{1-2q^2+q^4}}} \\ &= \frac{\frac{2q}{1-q^2}}{\sqrt{\frac{1-2q^2+q^4 + 4q^2}{1-2q^2+q^4}}} \\ &= \frac{\frac{2q}{1-q^2}}{\sqrt{\frac{1+2q^2+q^4}{1-2q^2+q^4}}} = \frac{\frac{2q}{1-q^2}}{\sqrt{\frac{(1+q^2)^2}{(1-q^2)^2}}} \\ &= \frac{2q}{1-q^2} \cdot \frac{1-q^2}{1+q^2} = \frac{2q}{1+q^2} = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \end{aligned}$$

$$\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \quad (57)$$

$\tan x = t/1$ denirse.



$$\sin x = \frac{t}{\sqrt{1+t^2}}, \quad \cos x = \frac{1}{\sqrt{1+t^2}}, \quad dx = \frac{1}{1+t^2} dt$$



$\tan x/2 = t,$

$$\sin \frac{x}{2} = \frac{t}{\sqrt{1+t^2}}, \quad \cos \frac{x}{2} = \frac{1}{\sqrt{1+t^2}}$$

$$\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2} = 2 \frac{t}{\sqrt{1+t^2}} \frac{1}{\sqrt{1+t^2}} = \frac{2t}{1+t^2},$$

$$\cos x = 2 \cos^2 \frac{x}{2} - 1 = 2 \frac{1}{1+t^2} - 1 = \frac{1-t^2}{1+t^2}$$

$$\frac{x}{2} = \arctan t \Rightarrow dx = \frac{2dt}{1+t^2}$$

$$\sin ax \sin bx = \frac{1}{2} [\cos(a-b)x - \cos(a+b)x]$$

$$\sin ax \cos bx = \frac{1}{2} [\sin(a+b)x + \sin(a-b)x]$$

$$\cos ax \cos bx = \frac{1}{2} [\cos(a+b)x + \cos(a-b)x]$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}, \quad (|x| < 1)$$

$$(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}, \quad (|x| < 1)$$

$$(\arctan x)' = \frac{1}{1+x^2}, \quad (-\infty < x < \infty)$$

$$(\operatorname{arc cot} x)' = \frac{-1}{1+x^2}, \quad (-\infty < x < \infty)$$

$$(\operatorname{arc sec} x)' = \frac{1}{|x|\sqrt{x^2-1}}, \quad (|x| > 1)$$